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LOW-FREQUENCY ROOM RESPONSES:

Part 2 – Calculation methods and experimental results

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Research Department, Engineering Division
THE BRITISH BROADCASTING CORPORATION

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Summary

The development of a relatively simple method for predicting low-frequency responses for near-rectangular rooms based on the summation of modal responses and their coupling coefficients for the source and receiver positions is outlined. Comparisons between calculated and measured responses show that many of the response features can be reproduced.

The question of optimum room shape is also addressed, and a method of illustrating the effects of room shape on the low-frequency modal distribution is developed. This uses a single criterion to plot contours of 'room quality' as a function of proportions.

Index terms: *Acoustics; acoustic treatment*

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1. INTRODUCTION

The background discussion to the work described in this Report is dealt with in a companion publication[1]. That document describes the way in which a room may be treated as a three-dimensional enclosure, the acoustic response of which is dominated by the eigentones which are the solutions of the wave-equation for the enclosure.

This present discussion will be confined to one method of prediction which is relatively simple to apply (in rectangular rooms), which is less dependent on exact and detailed material acoustic property information than some others and which has given theoretical results similar to those measured in practice.

The limitations and reasons for not pursuing the method further are also given in Ref. 1. They may be summarised as 'the quality of the best available acoustic data on materials does not justify further development'.

The predictions of some other aspects of the low-frequency acoustic behaviour in relatively small rooms are also discussed, together with detailed descriptions of a number of different experimental and field measurements.

2. RESPONSE PREDICTION USING MODE SUMMATION[2-5]

The coupling factor between a sound source or receiver in a room and a single axial (one-dimensional) room mode may be represented by a simple, single cosine function, if the coordinate origin is at one of the wall surfaces. At the positions on this function corresponding to 0, π, 2π, ... radians, the coupling factor will be a maximum, alternating in phase between 0 and π at adjacent maxima. At the intermediate positions, at odd multiples of $\pi/2$, the coupling factor will be theoretically zero; in practice, for damped systems, it will be very small. To extend this model to three dimensions and to include the effects of both source and receiver positions requires the inclusion of six cosine terms in all, for each mode. These may be combined into one equation, for the three principal coordinate axes of a rectangular coordinate system:

$$c_i(x,y,z)_S(x,y,z)_R = \Psi(S).\Psi(R) \qquad (1)$$

where c_i is the coupling factor for the ith mode,

S represents source quantities

and

R represents receive quantities.

The two distribution functions are similar to each other and can be represented by:

$$\Psi = \Psi(x,y,z)$$

$$= \cos\frac{n_x \pi x}{l_x} \cos\frac{n_y \pi y}{l_y} \cos\frac{n_z \pi z}{l_z} \qquad (2)$$

where n is the mode order,

l is the room dimension

and

x,y,z refer to the principal coordinate axes.

The magnitude of this coupling factor is proportional to the damping factor of the mode, in accordance with the common resonance response function. A third multiplication factor, related to the absolute difference between the two frequencies, will also apply, if the forcing frequency is not identical to the mode natural frequency.

The nett contribution of each mode is given by:

$$|p| = \frac{\epsilon_{x,y,z}\ \Psi(S)\ \Psi(R)}{2\omega_N k_N/\omega + i(\omega_N^2/\omega - \omega)} \qquad (3)$$

where ϵ is a scaling factor depending on the three-dimensional mode order,

ω_N is the mode natural angular frequency,

k_N is the damping factor of mode N

and

ω is the angular frequency at which the mode contribution is required.

The absolute magnitude of the total resonant field is also governed by the strength of the source, the size of the room and a number of media constants. In Ref. 2, an equation is derived giving an approximation for the instantaneous sound pressure level at a point in the resonant field. The interpretation of this and the extraction of usable expressions for this incorporation into a computer prediction program are given in the Appendix to this Report.

The total sound pressure at the receiving position due to all of the room modes is given by the summation:

$$|p| = \frac{\rho c^2}{\omega V} \sum_N \frac{\epsilon_{x,y,z}\, \Psi(S)\, \Psi(R)}{2\omega_N k_N/\omega + i(\omega_N^2/\omega - \omega)} \qquad (4)$$

where

V is the room volume,

ρ is the density of the medium,

and

c is the velocity of sound for the medium.

In addition to these resonance components, the direct sound may be significant. In Ref. 3, an expression is derived for the direct sound field at a distance from a simple source:

$$p = \frac{-i\omega\rho}{4\pi r}\, Q_0\, e^{i\omega\left(\frac{r}{c} - t\right)} \qquad (5)$$

where

Q_0 is the volume velocity

and

r is the radial distance from the source.

Again, the interpretation and conversion of this to a form usable in a computer program is given in the Appendix.

Equation 5 includes a term proportional to the frequency so that for a constant source volume velocity the sound level would also be proportional to frequency. Real loudspeaker sources do not behave in that way; the objective in loudspeaker design is usually to obtain a nominally constant output level. Thus, it can be concluded that a normal loudspeaker has an output volume velocity which is inversely proportional to frequency. (This is well-known anyway in the design of loudspeakers and could, in any case, be intuitively inferred because the normal type of loudspeaker is mass-controlled, at least at fairly low frequencies.)

Thus, the calculation methods used throughout this Report include this frequency-dependent volume velocity, of arbitrary overall magnitude. (In fact, the numerical value used was $1/2\pi f$. This gave predicted sound pressure levels in most cases around 60 - 70 dB, typical of those measured and illustrating that normal outputs are of about that value.)

The calculation method consists of first producing a data array for the room with the required source and receiver positions. For each mode, the natural frequency, combined source/receiver coupling factor and mode damping factor are stored. The total number of modes included is set by the upper frequency limit being considered. For most of this work, the upper frequency limit for the predictions was 200 Hz and the highest mode centre frequency included was 400 Hz (the typical values of damping factors used meant that each mode had very little effect at frequencies more than one octave from the natural frequency, particularly at high frequencies where the modal density is high).

Provision was made for the mode damping factors to be set individually. These were derived from the mean surface absorption coefficients and surface areas in the three orthogonal directions (see Appendix). The mean absorption coefficients were, in turn, derived from the reverberation times and room sizes. The absorption was assigned to all surfaces equally. In most practical cases, the floor and ceiling will have rather less than the average absorption coefficient and the walls rather more. The model used was capable of including these differences but the facility was not employed for the results given in this Report — there was little enough confidence in the values of the mean absorption coefficients (partly because of the uncertainty of reverberation time measurements at low frequencies*), without incorporating the additional refinement of uneven absorption distribution.

For the calculations, absorption coefficient values typical of acoustically-treated, studio-like rooms were used. For comparisons with real rooms, approximations to the actual reverberation time values were used, insofar as the 'reverberation time' at very low frequencies can be measured anyway. However, the detailed acoustic behaviour of real materials under these conditions is neither well documented nor easy to measure.

Following the derivation of the mode data, for each frequency, the contribution of each mode at that frequency was added to the total. The contribution of the direct sound component was then added. For computational convenience, the equations were expressed and the summations carried out in the form of real and reactive components.

A correction factor was introduced to represent the source frequency response. To model the usual kind of professional monitoring loudspeaker, a cut-off frequency of 50 Hz was used, with a 12 dB per octave slope below that frequency. No loudspeaker directivity factors were included — this is reasonable for frequencies up to about 300 Hz and is, in any case, implicit in the assumption made of the volume velocity factor.

* The reverberation time is a statistical concept, valid only for diffuse spaces. Even the measurement of reverberation time at low frequencies is an imperfect science.

Figs. 1 and 2 show an example of the results of such a calculation for an acoustically treated listening room with a volume of about 70 m^3. The overall response shown in Fig. 1**(a)** is the vector summation of about 200 modes, some of which are shown in Fig. 2. For comparison, Fig. 1**(b)** shows a measured response for the same conditions. In fact, the detail of the responses are so dependent on positions, even at low frequencies, that the measurements themselves are not entirely repeatable. There are clear indications that the overall characteristics of the response have been predicted, even if some of the frequencies and amplitudes are in error. Even the major characteristics at frequencies near to 200 Hz show significant similarities.

The objectives of this work and the limitations of the method are discussed in Ref. 1.

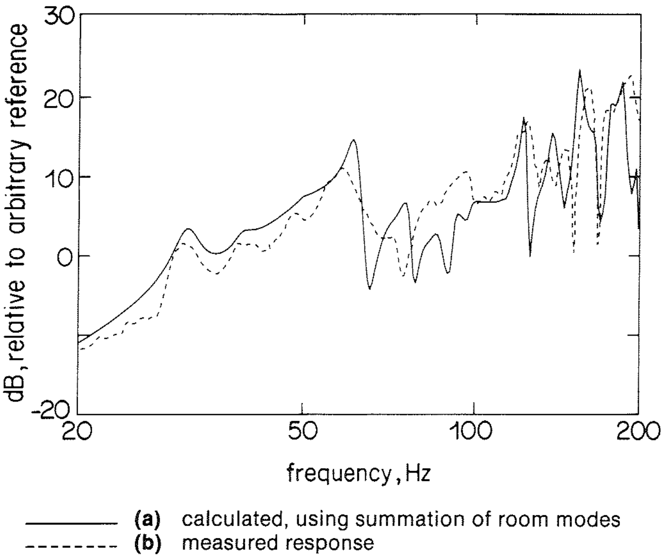


Fig. 1 - Low-frequency response in Listening Room 2.

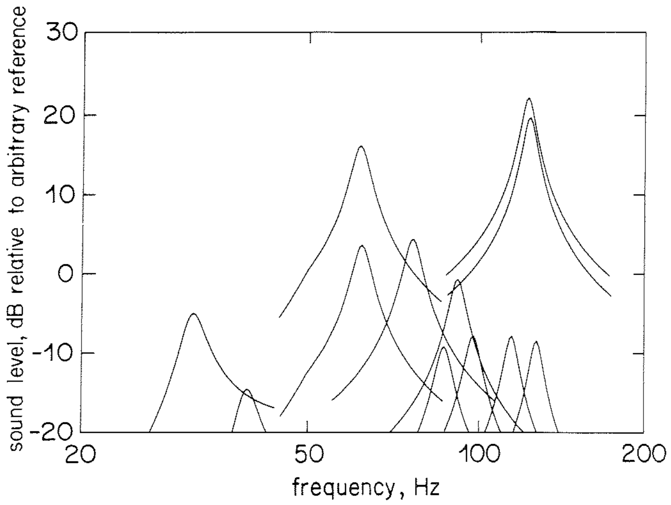


Fig. 2 - Sample selection of 12 modes, making up the response of Fig. 1 ***(a)****.*

A version of the response-prediction program which produced results in the form of one-third octaves was also developed. It was applied to those situations where high-resolution measured data were not available. The effective filters were sharp-edged, with infinite stop-band rejection at $\pm 1.12 \times$ centre frequency. This is not representative of practical filters and may have distorted the results in those cases where large irregularities occurred in the response prediction near to the filter band edges. It is probable that some useful refinements could be made to the one-third octave version. For the purposes of this work, such improvements were not considered justifiable.

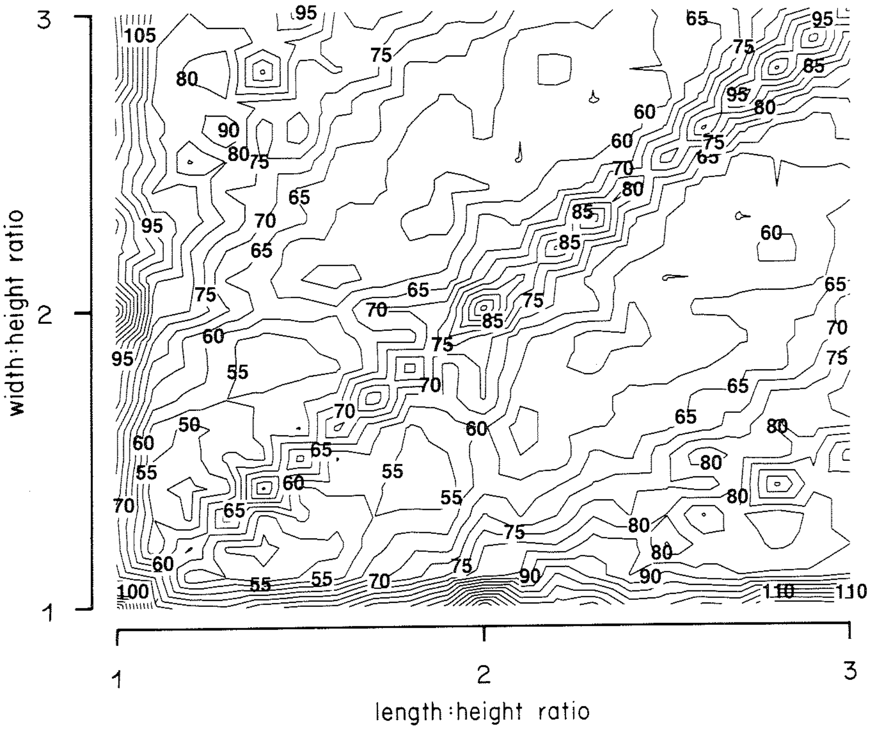
3. ROOM PROPORTIONS, AND 'GOLDEN RATIOS'

The problems created by isolated or sparse mode distributions are increased if the room proportions tend to cause the modes to cluster together. Based on the work described in Ref. 6, a computer program was developed to produce contour plots of some measure of room 'quality' for different proportions of a rectangular room.

In general, a rectangular room has four principal characteristic dimensional parameters — length, width, height and volume (any three of which are independent). Therefore, a fully general plot of the quality would be three-dimensional and difficult to portray on paper. By fixing one or more of these four parameters, the number of independent axes can be reduced. For example, if the volume is fixed, the independent parameters are reduced to two — the length/height and width/height ratios. The quality can then be portrayed as a two-dimensional contour map.

Each condition of room proportions requires the derivation and sorting of all mode centre frequencies (up to the limit set for consideration). These are then processed as required for the quality criterion. The resulting values can then be used as an input array to a suitable contour-plotting routine, to produce contour 'maps' which then display the quality criterion as a function of room proportions. The contour-plotting routine used in this work was based on that described in Ref. 7.

The example shown in Fig. 3 is for a fixed room volume of 200 m^3 and axes of length/height and width/height ratios, with a range of 1 to 3 for both (the result is therefore symmetrical about the diagonal from 1,1 to 3,3). This demonstrates the effects of room proportions without the bias towards



Figures are 10 × mean square mode spacing, higher 'quality' = lower mean spacing.

Fig. 3
Contour plot of room 'quality', for 200 m³ room, using mean square mode spacing for frequencies up to 120 Hz.

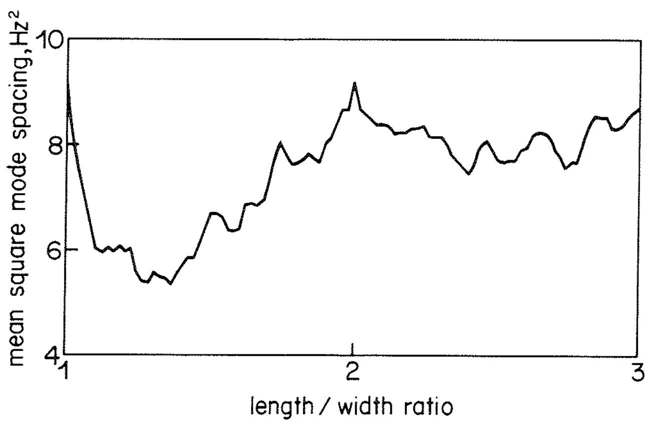
larger rooms which would arise if, for example, the room height was fixed and the volume allowed to increase with room proportions. The upper frequency limit was set to 120 Hz. This involved the processing of about 60 modes for each value of room proportion. The array of points used for the contour plotting was 20 × 20.

Several different quality criteria were investigated. The one most immediately obvious is the mean-square spacing of the mode frequencies. This gives greater weight to more widely-spaced pairs and is a common statistical measure of differences between sets of parameters. The data shown in Fig. 3 was calculated using this criterion. The numerical values indicate 10 times the quality value and are smaller for 'better' rooms. In this case, because the room volume is fixed, the height changes with floor proportions. The 'optimum' aspect ratios predicted by this means were 1.40 : 1.19 : 1 for nearly cubic rooms and 2.2 : 1.75 : 1 for rooms of which the width and length are both about twice the height. The height of 4.9 m for the nearly cubic room is, perhaps, unrealistic for a room of 200 m³. The proportions of about 8.2 × 6.5 × 3.7 m for the second room are more reasonable and fairly representative of practical rooms, at least the somewhat larger ones. The results also show the inferior quality of room proportions along the diagonal, corresponding to rooms with square floor plans, especially close to the integer multiples of the height (1 : 1, 2 : 2, 3 : 3). Rooms with proportions close to one axis (i.e. with one floor dimension nearly the same as the height) are also inferior. The 'better' rooms form a somewhat-curved, diagonal line, roughly parallel to the main diagonal but displaced from it, extending from about 1.2/1.4, through 1.75/2.2 and 2.2/2.8.

It could be argued that the spacing of the lowest modal frequencies is the most important factor and that the criterion should be weighted in that direction. However, the naturally more widely-spaced low frequency modes are implicitly weighted in that way by the mean square function.

Other possible criteria are the magnitude of the largest spacing between two adjacent modes and other powers of the mean mode spacing, such as first and fourth. In none of these cases did the optimum room proportions change very much from their mean-square criterion values. The conclusions about general areas also remained the same. There were, however, substantial changes in the appearance of the maps because of the different rates at which the criteria changed with changes in aspect ratios.

Fig. 4 shows a plot of the mean-square quality criterion for rooms of fixed height, fixed volume and variable ratio of length to width. The height was 3 m and the volume 100 m³. In this case, two parameters have been fixed and the resulting plot is a one-dimensional function of the floor shape. The region around 1.3 : 1 shows smaller values, with two local minima at about 1.27 : 1 and 1.36 : 1. The local maximum at 2 : 1 and the extended region of higher values over the whole range from about 1.7 : 1 upwards are also evident. The optimum at 1.36 : 1 represents a room 6.73 × 4.95 × 3 m, the overall size of which is quite consistent with typical control room sizes.



Values are mean square mode spacing,
higher 'quality' = lower mean spacing.

Fig. 4 - Plot of room 'quality', for 100 m^3 room of 3 m height, using mean square mode spacing for frequencies up to 120 Hz.

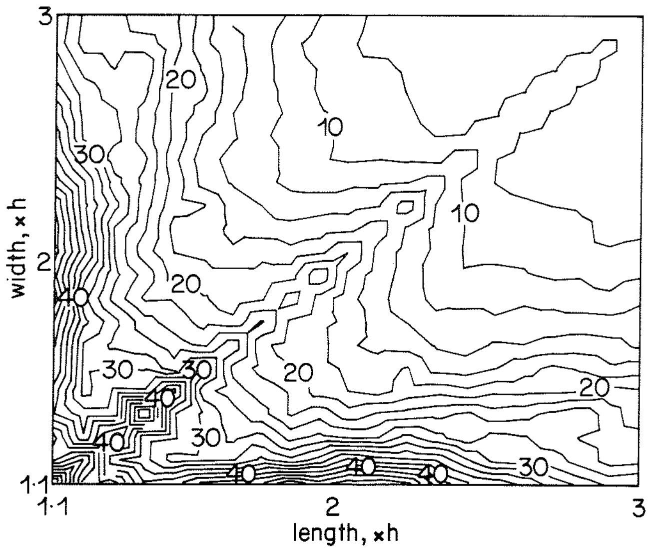


Fig. 5 - Contour plot of room 'quality', for 3 m high room, using mean square mode spacing for frequencies up to 120 Hz.

Finally, the beneficial effects of larger rooms are illustrated in Fig. 5 which shows a contour map of the mean-square quality criterion for rooms of fixed height (3 m) and variable volume. The same general structure as in Fig. 3 is evident but overlaid with a strong bias towards larger rooms (towards the 3 : 3 corner). The corner at 3 : 3 represents a room of 243 m^3. In this case, both axes begin at 1.1 × room height to avoid the dense contour distribution around the point 1 : 1 and to a lesser extent along both axes, caused not only by the poor room proportions but also the very small sizes of the rooms (27 m^3 at 1 : 1).

4. SOUND PRESSURE LEVEL DISTRIBUTIONS

An obvious combination of the two calculation methods described above is the prediction and plotting of sound pressure level distributions, for a single frequency and source position, on a two-dimensional section through a room. This process is, of course, subject to all of the limitations of the basic response prediction method but can still give results which are useful.

One demonstration of this method was in a case of a severe noise problem in an office beneath two chiller machines. The room dimensions were such that both of the horizontal axial fundamental modes were close to the operating frequency of the chillers, about 47 Hz. This gave rise to a pronounced stationary pattern of sound level distribution, with large peaks in two opposite corners. Although it was fairly clear what was governing the sound level distribution, as an example of the method and as confirmation of the problem, a sound level distribution pattern was calculated using the the method of mode summation. One problem was that the room was not rectangular, so that an approximation had to be made to allow for the rectangular 'cut-out' forming the door lobby. Fig. 6 shows the predicted sound pressure level distribution overlaid on the room plan. The high

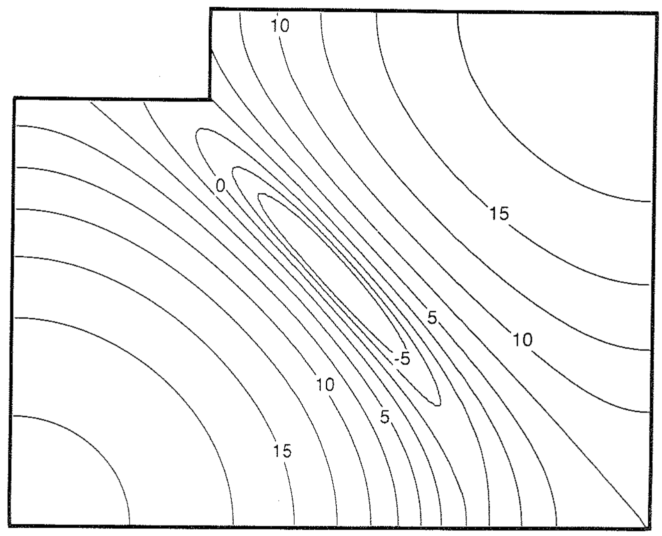


Fig. 6 - Sound pressure level distribution prediction, calculated using mode summation.

sound levels in the two opposite corners are clearly visible.

As an illustration of the potential reliability of more complex calculation methods, the same problem was solved using a well-known Finite Element Analysis (FEA) package. In this case, the actual shape of the room was used rather than the rectangular approximation. For simplicity and economy of calculation, it had been based on a two-dimensional model because it had reasonably been assumed that the main frequency of interest was below the first floor-ceiling resonance, and the sound field would, therefore, have no significant height-dependent variations. Despite some efforts to alter the assumed position of the vibration source, the FE results did not produce a prediction which was correct everywhere.

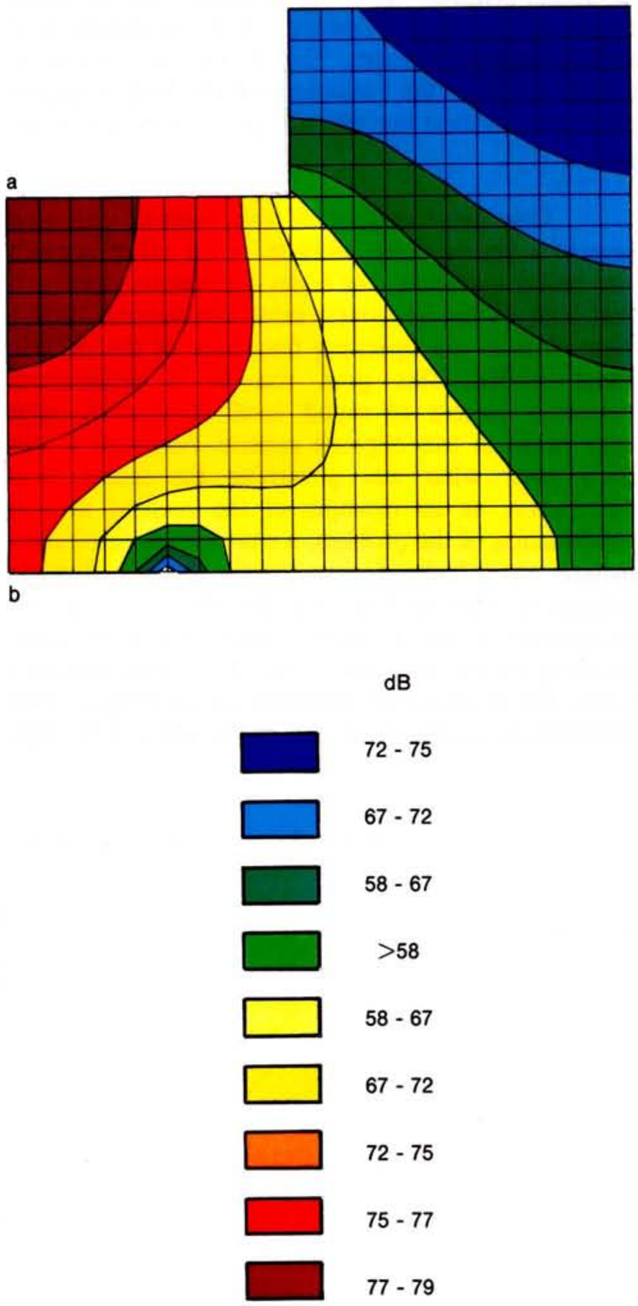


Fig. 7 - Sound pressure level distribution prediction for the same room as Fig. 6, calculated using Finite Element Analysis.

Fig. 7 shows a typical example. The major error was the predicted maximum near to the door lobby, at 'a'. In fact, that maximum was in the adjacent corner, 'b', more like the prediction from the simpler method. This example serves to demonstrate again that the problems of accurate specification of the model are paramount and are not necessarily overcome by very complex and expensive processing methods.

5. EXPERIMENTAL MEASUREMENTS

5.1 Identification of modal frequencies

A series of early experiments on the measurement of mode frequencies was carried out in an acoustically treated listening room. The room was close to rectangular in shape, of gross internal dimensions 5.54 × 4.44 × 3.10 m. The reverberation time characteristic of the room was as shown in Fig. 8. The objective was to discover the magnitude of the effects of typical acoustic treatment on the frequency and distribution of the room modes.

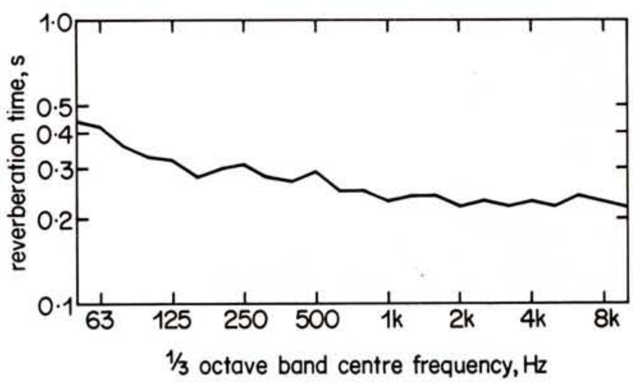


Fig. 8 - Reverberation time characteristic in BBC Research Department Listening Room 2.

The room construction consisted of masonry walls and concrete ceiling slab, with a wooden floor on joists and suspended false ceiling. The acoustic treatment on the walls consisted of modular absorbers of a complex and obsolescent type covering most of the available surface between 0.75 m and 1.75 m from the floor. These modules were composed of two layers of 30 mm fibrous material (bonded acetate fibre), the front one being sandwiched between two sheets of perforated hardboard, over a 60 mm deep membrane absorber, using bituminised fibre sheet ('roofing felt') as the resonant panel. The wall surfaces up to 0.75 m on the two long sides were 100% treated with low-frequency panel absorbers (bonded hardboard and roofing felt). The remaining wall surfaces were almost bare, a small percentage being treated with panels of 30 mm thick mineral wool. The floor was covered with standard woven carpet on felt underlay. The structure of the false ceiling at a height of 2.75 m consisted of hardboard panels with inset fluorescent light fittings on a timber framework.

In the sense that the reverberation time was typical of contemporary installations, the mechanisms of absorption were the same and the distribution of the treatment was reasonably uniform, the room was considered to be sufficiently representative of current practice to be used for this experimental work, despite the unusual nature of the main acoustic treatment. It will be seen that the differences observed between actual and theoretical modal frequencies were, in any case, only a relatively small percentage.

The excitation and identification of the lowest frequency modes in such a room is trivial, even though the damping factors are relatively large. The modes may be selectively excited simply by tuning a signal source to their respective maximum responses, with an arbitrarily positioned loudspeaker. Just a little care is required to ensure that neither the loudspeaker nor the listener/microphone are close to pressure nodes. The sound pressure distribution of any of these modes can readily be appreciated (and measured) by movement of the observer (or the microphone) around the room. However, at frequencies higher than about the second harmonic of the width in such rooms, the modes overlap to the extent that they are not directly identifiable individually. It is then necessary to attempt to excite them selectively by using multiple loudspeakers of appropriate relative phases. For example, if the second harmonic of the room length coincided with the fundamental of the width (an undesirable condition anyway but one which will serve to illustrate the point), the latter could be excited selectively, using two loudspeakers in antiphase on opposite sides of the room at one end. In that way, the length resonance would be excited less because of the conflicting phase of the loudspeakers coupling to the same phase of the resonance. For the width resonance, the two coupling factors would be in-phase, the electrical phase inversion being consistent with the acoustic phase difference.

Using such methods and up to six loudspeakers, 14 of the modes up to the third harmonic of the length were fairly clearly identified. The results are shown in Table 1, together with the theoretical mode centre frequencies. For the calculations, the effective room length and width were taken to be to the structural surfaces, rather than to the visible surface of the acoustic treatment. The basis for the height in the calculations was less clearly obvious. The structure of the false ceiling was relatively light and unperforated, with hardboard panels over 250 mm airspace. Such panels are likely to have fundamental resonance frequencies of about 60 - 80 Hz. Therefore, for frequencies above about 100 Hz, the false ceiling structure would behave as a rigid surface. Below that, and especially around the resonance frequencies, it would be likely to have a relatively low impedance. In contrast, the suspended floor, although probably having similar resonant frequencies, was much more massive and likely to present a high acoustic impedance. Therefore, for these frequencies (which were mostly below 100 Hz) the effective height of the room was taken to be that from the suspended floor to the structural ceiling slab. This is at least partially justified by the small error, −0.18%, in the first 'height' mode at 55 Hz.

Table 1: Measured room modes.

Mode order (HWL)	Frequencies, Hz		
	Calculated	Measured	% error
001	31.00	31.30	−0.96
010	38.70	38.00	1.80
011	49.50	47.10	4.84
100	55.50	55.60	−0.18
002	62.00	57.80	6.77
020	77.30	76.60	0.90
102	83.20	81.40	2.16
021	83.30	82.20	1.32
112	91.70	91.70	0.00
003	93.00	93.20	−0.21
120	95.20	94.40	0.84
022	99.10	99.50	−0.40
013	100.7	102.7	−1.98
103	108.3	107.7	0.55

HWL is mode order, in multiples of height, width, length.

Overall, the normalised differences are generally less than 2%, although two modes differ by 5 - 6%. The mean normalised difference is 1.1% and the normalised standard deviation is 2.2%. There is a slight tendency for the measured frequency to be lower than the calculated, corresponding to an acoustic size slightly greater than the physical size.

The largest errors occur at frequencies between about 45 and 85 Hz when the modes are functions of length or width only (i.e. 011 and 002 particularly and, to a lesser extent, 021). It may be that these errors represent the effect of panel resonances in the low-frequency acoustic treatment on the walls, which probably occurred at 60 - 80 Hz. It is likely that this treatment will appear as a compliance at frequencies below the resonance, effectively increasing the apparent size of the room. This is reinforced by the opposite sign of the error for similar modes at higher frequencies (i.e. 003, 022 and 013). However, the data is limited so the statistical evidence for these effects is hardly significant.

The calculations were based on the gross room internal length and width rather than the actual dimensions inside the acoustic treatment, the thickness of which represented an average of about 2% of the room width and length. Thus, the acoustic effect of the treatment has been shown to be more than enough to overcome its physical size, leaving a room which is, acoustically, just about the same size as its solid external boundaries.

5.2 Frequency response predictions in an acoustically-treated Listening Room

The same room as used for the study of modal frequencies in Section 5.1 was also used as the basis for comparison of predicted and measured frequency responses, based on the method of Section 2. For this room, with its impervious but fairly light false ceiling, the question of the appropriate height is significant and the answer not immediately obvious. The original computer prediction program did not permit the effective height of the room to be expressed as a function of frequency. To avoid the complexity of substantial modifications to the software, the majority of predictions were made for the false ceiling height. These are likely to be justifiable at frequencies above about 80 Hz and also at frequencies significantly below that of the first height mode in the larger room (55.6 Hz).

Fig. 1 (*on page 3*) shows the results for a source at 1.13, 1.05, 1.00 and a receiver at 2.68, 2.20, 1.23, the origin of the coordinate system being at the front, lower left-hand corner of the room. The room size, in the same coordinate system, was 4.44, 5.54, 2.75. A second condition, with the same source position and a receiver at 3.47, 1.61, 1.42 gave the results shown in Fig. 9. Both of these figures are likely to be erroneous in the frequency range 50 - 80 Hz because of the height effect described above. The results generally support that likelihood.

To illustrate the effects of room height, Fig. 10 shows the responses for the same conditions as Fig. 1, but with the room height set at 3.1 m, for a frequency range only up to 100 Hz. It shows that the prediction is nearly identical to that of Fig. 1 for frequencies up to about 50 Hz and is arguably more accurate between 50 and 80 Hz. Fig. 11 is the low-frequency response for the same height, plotted for the second position. In this case, the prediction has been plotted shifted down in frequency by a factor of 1.05. The closeness of fit (for a total of seven distinguishable features below about 65 Hz, several of which do not involve height modes) can now be seen to be remarkable, but the origin of this constant 5% factor remains a mystery. It is as though the velocity of sound had been 328 m/s, a value which would occur at an air temperature of −5C.

It is clear that the overall responses can be predicted with a reasonable degree of accuracy, provided that the gross effects of room structure are taken into account. It is also clear that the frequencies of even the lowest, isolated modes are slightly in error. This is especially evident in Fig. 9. That this is due to the errors in isolated mode centre frequency, caused by differences between the theoretical and real conditions, seems unlikely in view of the relatively small values of those differences. Fig. 9 also shows the mode centre frequencies measured in isolation, from Table 1, in the same room. It is clear that the modal centre frequencies appear to be shifted when several occur simultaneously, a finding reported by earlier work[8].

Making more accurate allowances for material and structural properties of the room boundaries is a subject which would require further study and is

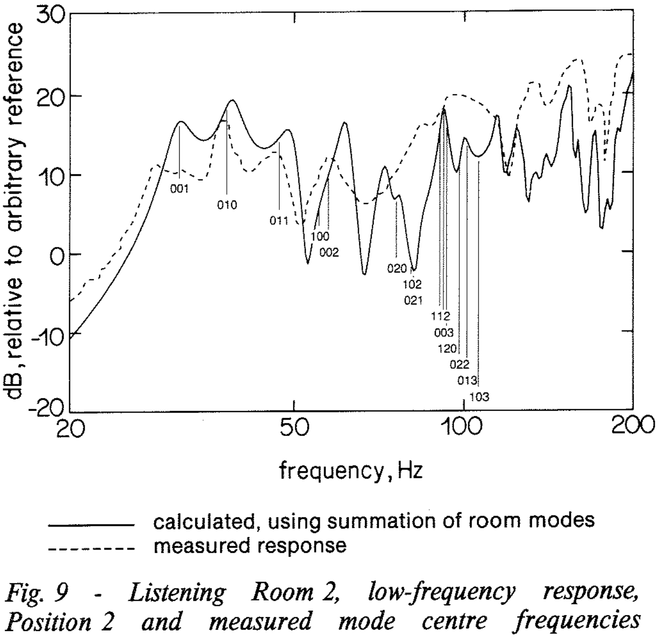


Fig. 9 - Listening Room 2, low-frequency response, Position 2 and measured mode centre frequencies (from Table 1).

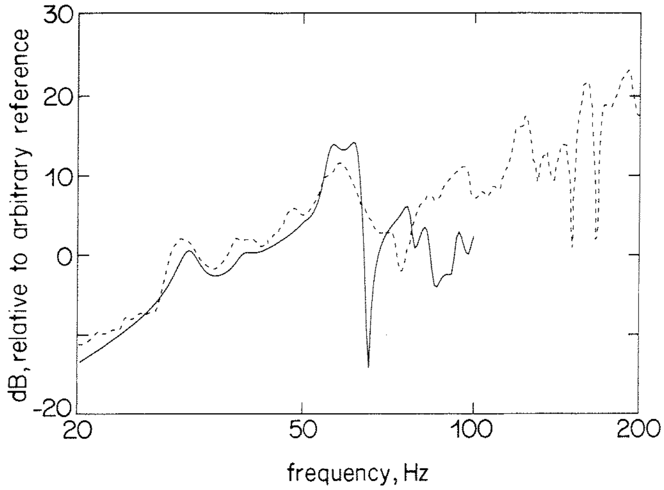


Fig. 10 - Listening Room 2, low-frequency response, Position 1, full room height.

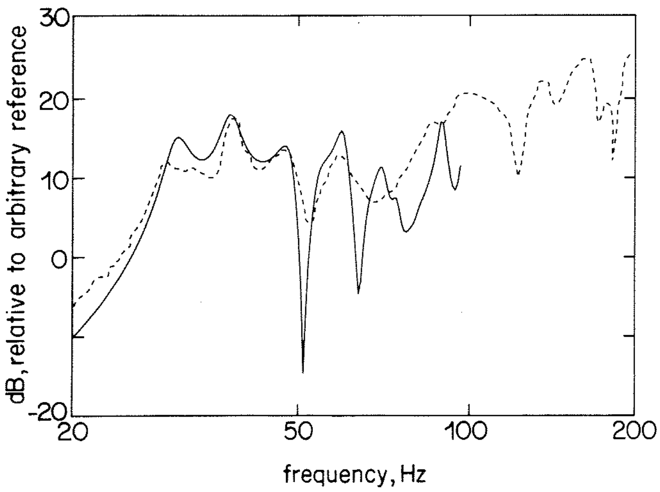


Fig. 11 - Listening Room 2, low-frequency response, Position 2, full room height.

outside the scope of this present work. It would require the acoustic properties of resonant structures and their effect on the mode sound level distribution patterns to be quantified, for both normal and oblique angles of incidence. In most cases, it is likely that the simple, analytic approach to rectangular rooms, as adopted for the present work, would be be inadequate for the more detailed study and that numerical methods, such as Finite Element Analysis, would be required to calculate mode distributions.

5.3 Glasgow Studio 1

The completion of a new control cubicle for Glasgow Studio 1 (in 1985) was the event which first brought these low-frequency problems into prominence and initiated the work reported here and in Ref. 1. In the design, great efforts had been made to ensure that the acoustic environment was symmetrical — to give optimum stereophonic image qualities. The first subjective evaluation indicated that the sound quality of the low-frequency part of the spectrum was unsatisfactory. In particular, with the loudspeakers at their design spacing of 2.4 m, the response amplitude in the 50 and 63 Hz bands was judged to be low. It was also discovered (by the users) that the effective response depended on the loudspeaker spacing. In fact, what had been (re)discovered was the dependency of response on the geometry and internal arrangements of the room. Measurements were made of the effective response, for both loudspeakers simultaneously, as they were moved along a line through their 'normal' positions. At all times, the symmetry of the loudspeaker disposition relative to the room and the operator's position was maintained. The range of loudspeaker spacings used was from about 1.7 m to 4.3 m. The range of sound pressure level values obtained at 50 Hz was 10 dB, with the minimum value occurring at a spacing of 3.0 m. At 63 Hz, the range was 11 dB, with the minimum at a spacing of 3.7 m.

At that time, there was no information about the predictability of these effects (although the problems had been recognised within the BBC for many years). However, it was fairly self-evident that the second axial mode of the room width and the position of the loudspeakers, in relation to that modal pressure distribution, were dominant factors. One of the reasons why these ideas had been less appreciated than they might have been was a (false) notion that they were relatively narrowband effects — too narrow to be subjectively significant. In fact, with the quantity of acoustic treatment used in contemporary control rooms, the damping factors are relatively high and the effects comparatively wideband. In Glasgow Cubicle 1, the second axial mode of the width is at about 59 Hz and the loudspeakers would be located at the critical pressure nodes at a spacing of 2.9 m. This is close to their design spacing. Most acoustic measurements are carried out in standard ISO one-third octave bands, the crossover frequency nearest to 59 Hz being at 56 Hz. Thus, the measurement technique is not able to resolve such differences, although it is fairly well-matched to the bandwidth of the effects.

To illustrate the potential predictability of these effects, the ⅓rd-octave based computer program was modified to include shifting of the source position. The results were compared with the measurements made in 1986. Figs. 12 and 13 show the comparisons for the third octave bands from 32 to 200 Hz. The results have been displaced in 20 dB steps to separate them.

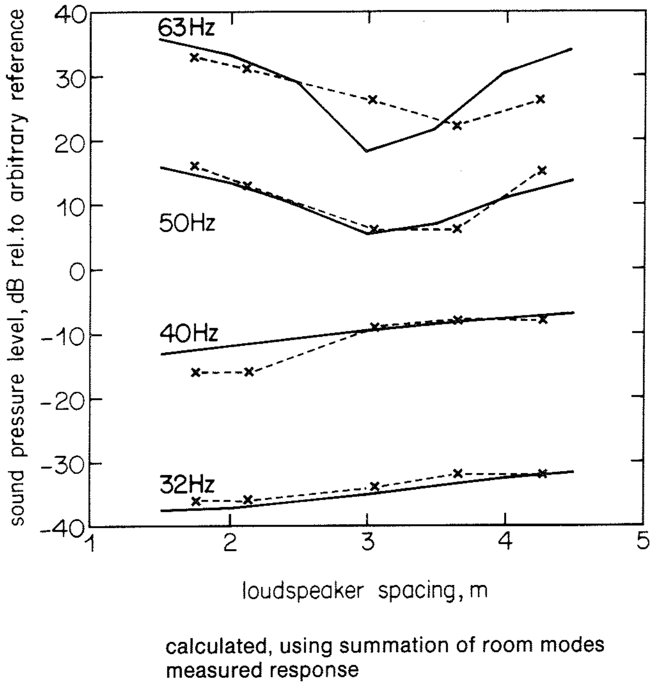


Fig. 12 - Glasgow Cubicle 1, response variations with loudspeaker position in one-third octave bands, 32 Hz to 63 Hz.

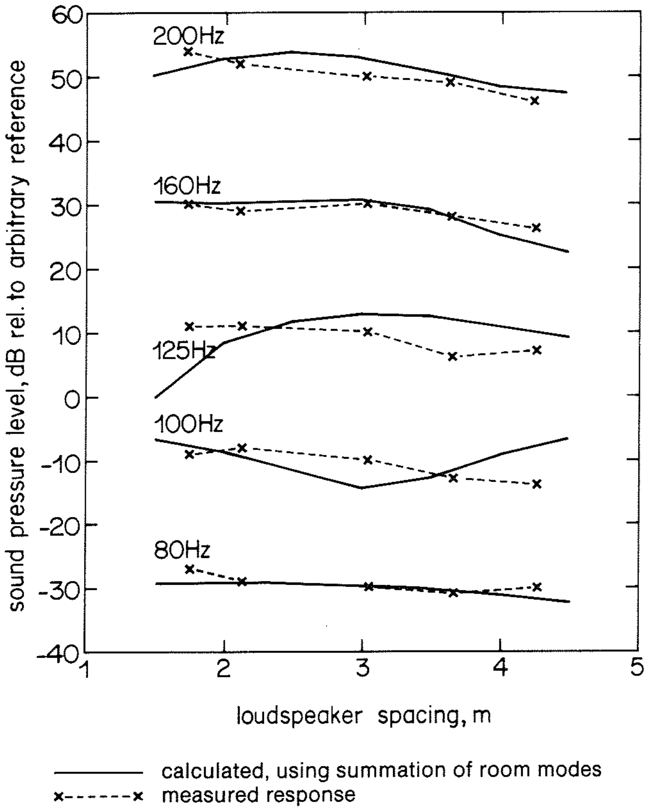


Fig. 13 - Glasgow Cubicle 1, response variations with loudspeaker position in one-third octave bands, 80 Hz to 200 Hz.

Also, the absolute levels have been adjusted by arbitrary amounts because the original data had been normalised and had therefore lost the reference levels. In any case, the absolute setting of the amplifiers, the differences in individual loudspeaker responses and the lack of absolute levels from the prediction program make absolute comparisons impossible. One of the problems arising from one-third octave predictions with infinitely sharp filters is the sensitivity of the results to small frequency shifts. This can take a feature from one band into the next. In this case, the data was available only in the one-third octave form.

The original objective of this work, as triggered by the early results from Glasgow, was to identify the mechanisms responsible. It is clear from Figs. 12 and 13 that, despite some large discrepancies, there are significant similarities between the measured and predicted effects and that the mechanism has not only been identified but to some extent quantified. To illustrate the reliability of the prediction process for this case, Fig. 14 shows the prediction for the 63 Hz band with room turned round through 90°, that is, the width and length interchanged. It is compared with the measured response function for the real room. Whereas the results in Figs. 12 and 13 show similar shapes for the same frequency bands, the prediction in Fig. 14 is clearly of the wrong character.

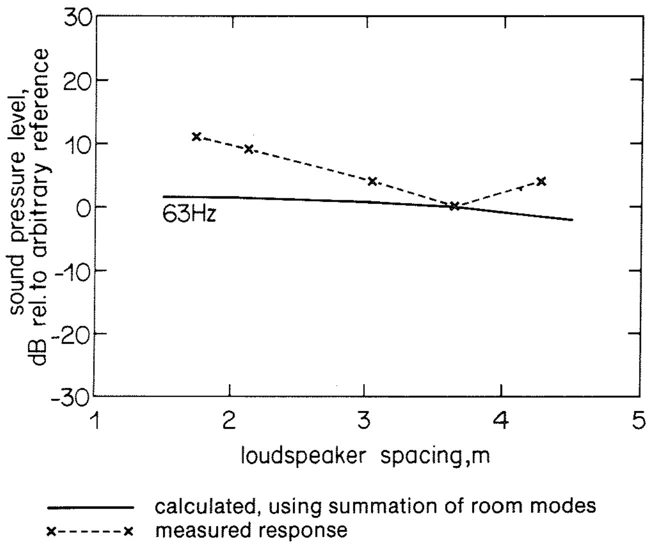


Fig. 14 - Glasgow Cubicle 1, response variations with loudspeaker position in one-third octave bands, 63 Hz, using wrong room data.

5.4 Maida Vale Studio 1

An investigation was carried out in the control cubicle of Maida Vale Studio 1 following complaints about perceived sound quality. As part of this, the low-frequency steady-state response was measured for the two loudspeakers at the operator's position. The gross room volume, to the structural ceiling slab, was about 79 m^3. Fig. 15 shows a sketch of the layout, and Fig. 16 the one-third octave frequency responses.

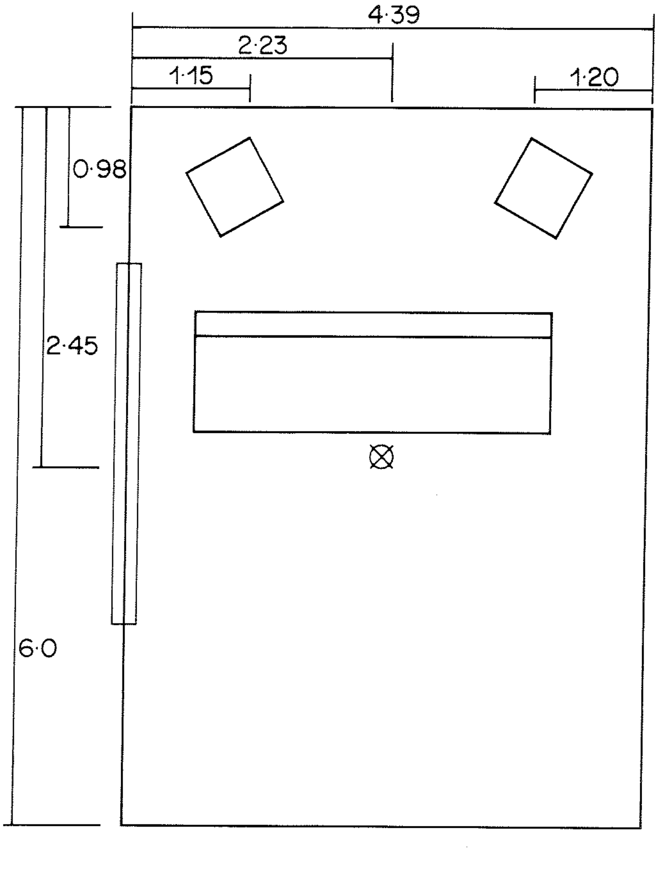


Fig. 15 - Sketch showing Maida Vale Cubicle 1 layout.

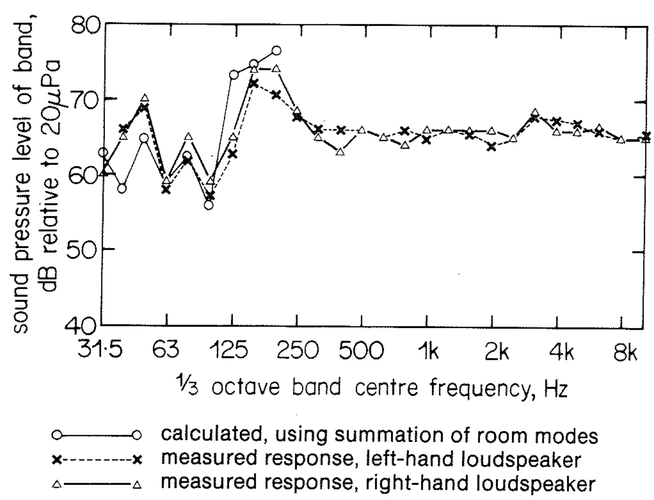


Fig. 16 - Maida Vale Cubicle 1 one-third octave frequency response.

The room was, geometrically, almost perfectly rectangular, with nearly symmetrically disposed loudspeakers and listening positions. The measured results show that this geometrical symmetry had also resulted in acoustical symmetry, with the responses from the two loudspeakers being essentially identical to each other. The main features of the measured responses are very large irregularities at frequencies up to 315 Hz, with emphasis at 50 Hz and 160 - 200 Hz and a broad, deep dip from 63 Hz to 125 Hz. The predicted response, which is also essentially identical for the two loudspeakers, echoes the general form of the measured responses, but does show some significant differences, particularly in the 40 and 50 Hz bands.

An attempt to correct these responses by an additional low-frequency loudspeaker failed to make a significant subjective improvement. The main reason for this was probably that the irregularities extend upwards in frequency to a region where a separate low-frequency source is not a credible option. If it were possible to modify responses in the vicinity of 200 - 250 Hz by relocation of the low-frequency loudspeaker — something which is itself unlikely because of the high modal density at those frequencies — it would be audible as a distinctly separate source. Such frequencies are well above the limit where the listener can be persuaded that the low-frequency and the mid/high frequency sources are co-sited.

5.5 Edinburgh Studio 1

An investigation was carried out in the control cubicle of Edinburgh Studio 1, also following complaints about perceived sound quality. As part of this, the low-frequency steady-state response was measured for the two loudspeakers at both the operator's position and at the producer's desk immediately behind. The gross room volume, to the structural ceiling slab, was about 126 m^3. Fig. 17 shows a sketch of the layout and Figs. 18 and 19 the one-third octave frequency responses for the operator's and the producer's positions respectively. Once again, the room was almost perfectly rectangular, with symmetrically disposed loudspeakers and listening positions. Figs. 18 and 19 show that this geometrical symmetry had also resulted in acoustical symmetry, with the responses from the two loudspeakers being very similar to each other at both listening positions, with only minor localised deviations. The main difference was an anomalous peak of about 8 dB amplitude, at 160 Hz, for the right-hand loudspeaker measured at the producer's position. Furthermore, the overall responses were fairly uniform, especially at the producer's position. These results are in contrast with most other areas, some examples of which are shown in other sections of this Report. For that investigation, it was concluded that there was no obvious reason why these low-frequency responses could be responsible for any difficulties with the sound quality.

Figs. 18 and 19 also show the predicted one-third octave results. Because of the geometrical symmetry, the predictions were the same for the two loudspeakers. In Fig. 18, the predicted response at the operator's position shows some significant similarities with the measured ones. Except for the peak at 125 Hz, the prediction consists of identical features, albeit with some differences of amplitude. For the results at the producer's position, Fig. 19 shows a much less accurate prediction. The predicted peak in the 40 Hz band and the dip in the 50 Hz band are not present in the measured result to any significant degree. The large predicted peak at 125 Hz appears to occur, at a lesser magnitude, at 160 Hz. This may be due to the sharpness of the calculated one-third octave filters. However, the slight increase in the values of the next lower bands in both cases (100 and 125 Hz respectively) suggests that the bandwidth of the effect is not that narrow. Overall, this appears to be a rather poor prediction.

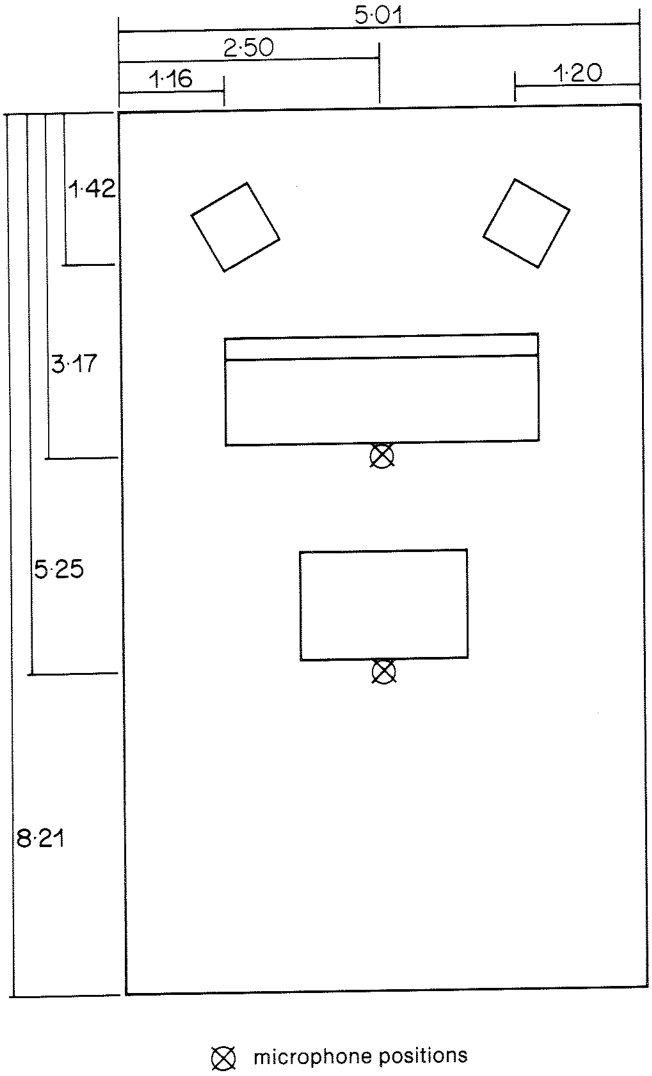


Fig. 17 - Sketch showing Edinburgh Cubicle 1 layout.

5.6 Television Centre, Dubbing Theatre 'W'

The effective frequency response in this new facility showed a strong reinforcement of the 63 Hz one-third octave frequency band at the mixing desk operator's position. In fact, this excessive response extended over the whole width of the desk. Fig. 20 shows a sketch plan of the room layout and Fig. 21 shows the measured and predicted frequency responses for the operator's position at the mixing desk.

It was clear that the measured peak at about 63 Hz was caused by the positions of the loudspeakers and the desk — both being located near to pressure antinodes of the third harmonic of the room length, which had a theoretical frequency of 69 Hz. The only other mode with a centre-frequency in that region of the spectrum was the second harmonic of the room width, at a theoretical frequency of 72 Hz. The loudspeaker positions, again approximately at the pressure nodes, ensured that virtually no coupling to the transverse mode occurred. Thus, the response was dominated by the longitudinal mode, with emphasis by two additive effects. The difference between the measured and calculated frequencies for this mode was

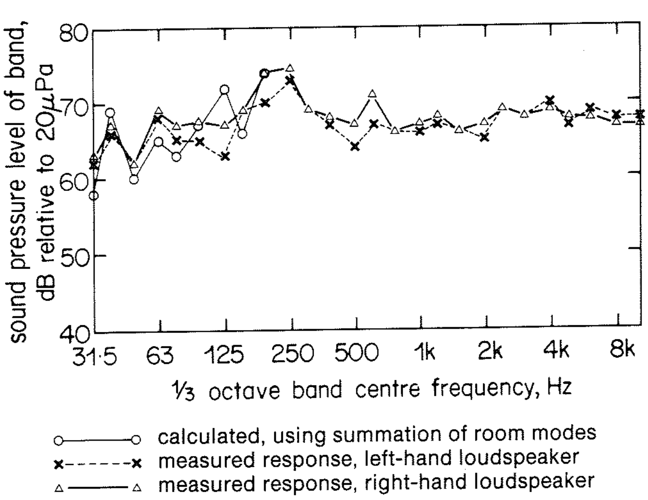


Fig. 18 - Edinburgh Cubicle 1 one-third octave frequency response at operator's position.

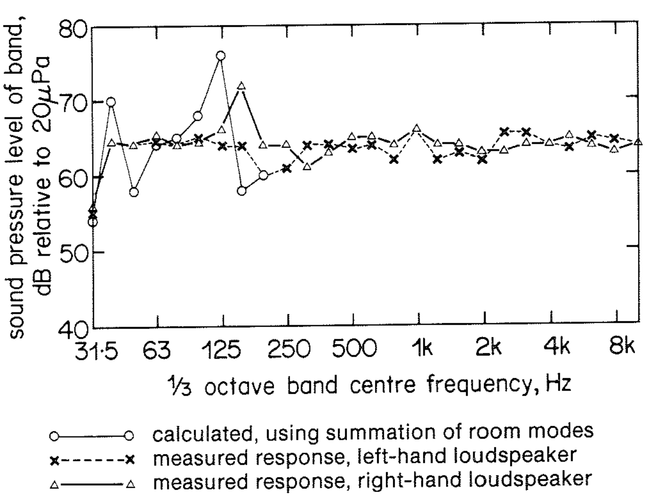


Fig. 19 - Edinburgh Cubicle 1 one-third octave frequency response at producer's position.

large. The room length gives a theoretical frequency for the third harmonic of 69 Hz, even ignoring the small reduction due to the angle of the doorway.

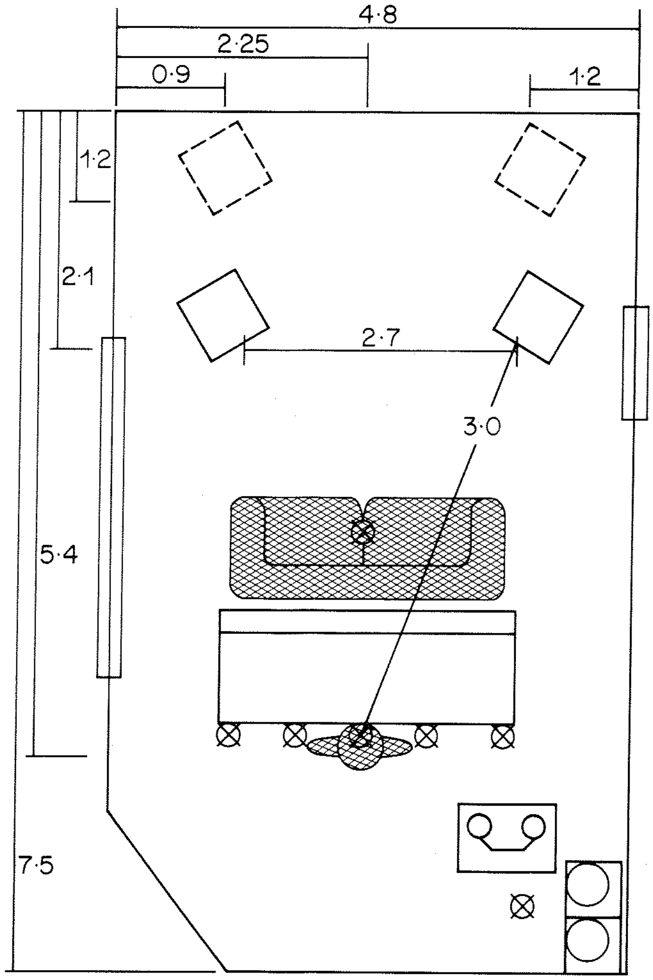


Fig. 20 - Sketch showing Dubbing Theatre 'W' layout.

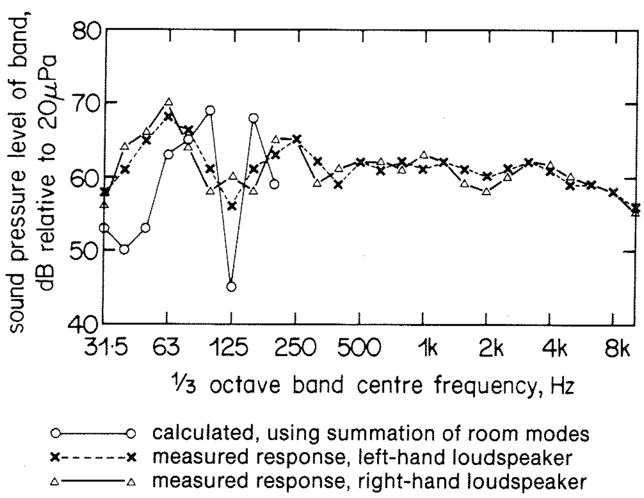


Fig. 21 - Dubbing Theatre 'W' one-third octave frequency responses at mixing desk.

However, the mode was clearly identified at 63 - 65 Hz, a difference of about 8%. The construction of the room was quite different to anything else investigated in the context of this Report, consisting of a proprietary design using a prefabricated steel shell incorporating the acoustic treatment in a thickness of about 100 mm. The acoustic length of the room appeared to be about 8.1 m, rather than the 7.5 m actually measured. That the steel panels could be sufficiently resonant at frequencies around 65 - 70 Hz to account for these differences is not very surprising.

Overall, the prediction is significantly worse than the others described in this Report (although the deep dip at 125 Hz is predicted). Despite much effort, no convincing reason could be found for the failure of the calculation method to predict the general and rather broad peak measured in the 63 Hz band, notwithstanding its fairly obvious cause. The computer model included only one source, whereas the measured results are for both loudspeakers operating. As described above, with in-phase excitation on both sides of the room, there would be cancellation of some 'width' modes. However, eliminating all of the width modes from the prediction did not produce a better result. This remains as one of the problems to be investigated in any extension of the work using one-third octave bands.

The normal use of the room required the loudspeakers to be movable in order to accommodate a significant number of production team members, in addition to the normal sound operational complement. Furthermore, the space requirements did not permit additional low-frequency loudspeakers. The normal operational procedure was to carry out most of the preliminary mixing and adjustment with the loudspeakers positioned as shown in Fig. 20. The final 'mixdown', with the larger audience, was usually done with the loudspeakers nearer to the end wall.

Measurements were made of the changes in frequency response for progressive changes in loudspeaker distance from the wall, from 2.2 m to 1.0 m in steps of 200 mm. The results are shown in Fig. 22, with consecutive plots displaced by 10 dB for clarity. The progressive reduction in the magnitude of the peak at 63 Hz can clearly be seen. At a distance of about 1.2 m, the effect of the pressure antinode near the listener was almost completely cancelled by that of the pressure node at the speaker position, producing an overall response which was reasonably uniform.

The practical limitations imposed by the demands on this space did not permit any permanent improvements to be recommended. Instead, the established working practice, of carrying out most of the preliminary mixing with the loudspeakers in the forward position and leaving the adjustment of overall balance until later, was confirmed and reinforced by these objective results. More recent work, if it had been available at the time, might have suggested additional low-frequency loudspeaker positions in, for example, the ceiling.

Whether the room would have been 'better' with some contribution from the transverse mode is a debatable point. It would have introduced response variations with lateral position. Without it, the response was at least consistent across the width of the sound mixing desk. This illustrates the conflicting factors which have to be balanced in such cases.

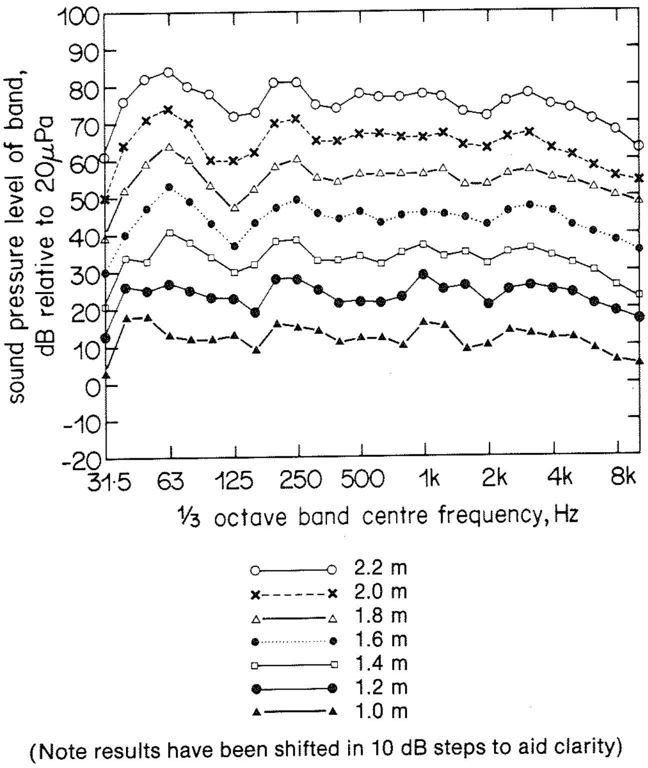


Fig. 22 - Dubbing Theatre 'W', one-third octave frequency responses, for different loudspeaker positions, 2.2 m to 1.0 m from end wall.

5.7 Pebble Mill, Studio 2

An investigation was carried out in the control cubicle of Pebble Mill Studio 2, again following complaints about perceived low-frequency sound quality. Fig. 23 shows a sketch of the layout for the two loudspeakers in their normal positions. The room was nominally rectangular, with symmetrically disposed loudspeakers and listening positions. At the rear of the room and offset to one side was an annexe, connected to the main room by an opening about 1.6 m wide. Such a geometry is, strictly, beyond the capabilities of the simple prediction method. However, during the investigation it had been found that the annexe contributed little to the room response at frequencies between about 50 Hz and 125 Hz; probably because the inertance of the aperture, to some extent, behaved as a reflector at those frequencies. Both at lower frequencies and at higher frequencies, the mode distributions did include the annexe and were, therefore, of complicated shapes. Consequently, any predictions based on rectangular rooms are likely to be invalid for those frequency bands. Fig. 24 shows the one-third octave frequency responses measured at the operator's position for the

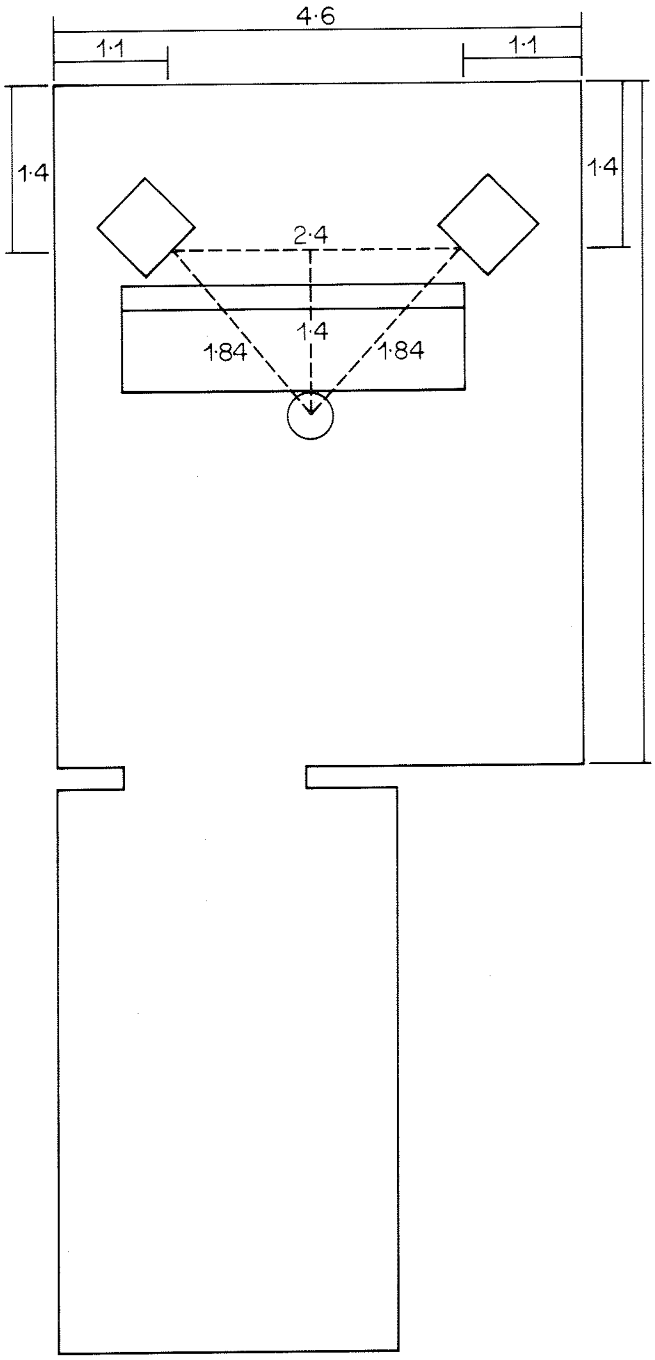


Fig. 23 - Sketch showing Pebble Mill Cubicle 2 layout.

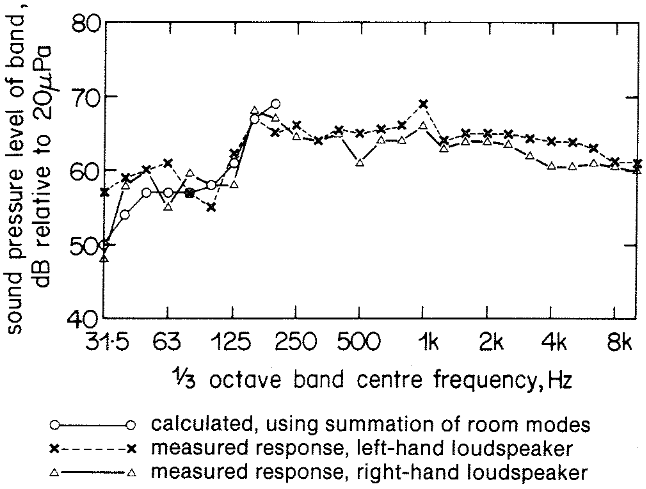


Fig. 24 - Pebble Mill Cubicle 2 one-third octave frequency response.

two loudspeaker locations, together with the predicted response from the simple model. In all cases, the poor low-frequency response is obvious. Considering the limitations and reservations, the prediction does echo some of the general features of the measurements.

Fig. 25 shows the frequency responses measured at the operator's position for both main loudspeakers at the same time, and with the use of an additional low-frequency loudspeaker and experimental 'cross-over' filter. The cross-over frequency was 100 Hz. The additional loudspeaker was positioned in the normally-unused space behind the mixing desk and at a point, determined by experiment, to generate the most uniform overall response. It is clear that the effective overall room response has been made more nearly 'flat', mostly within ± 2 dB for all frequencies below about 500 Hz.

Unfortunately, the subjective response from the users' representative was that the room problem had

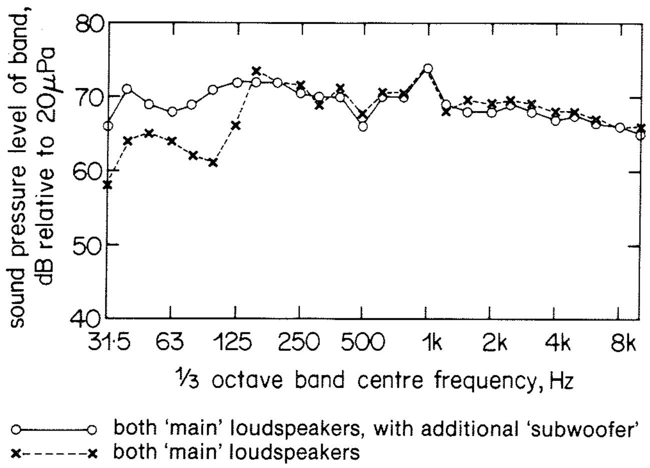


Fig. 25 - Pebble Mill Cubicle 2 one-third octave frequency responses, with additional low-frequency loudspeaker.

not been solved. This may illustrate that the room quality, even at low frequencies, cannot be quantified objectively or modified in this way. Alternatively, it could indicate that other problems were dominant.

5.8 New loudspeakers

The regular use of separate low-frequency loudspeakers would suggest that it might be beneficial to reconsider the design requirements for control room monitoring loudspeakers. In the past, all BBC monitoring loudspeakers have been single assemblies covering the whole frequency range. Their size is almost entirely determined by their low-frequency response limit (with some secondary consideration of power-handling ability). In many cases, the space available in a control room determines the maximum permissible size and, thus, the acoustic performance of the loudspeakers. By raising the low-frequency limit by an octave or more, the size could be reduced very significantly, without compromising the other parameters. This could permit the use of 'better' loudspeakers in areas hitherto considered too small.

These smaller loudspeakers would require the addition of a separate bass unit to extend the performance down to the necessary low-frequency limit, which could be dependent on the circumstances. For example, a mobile control unit, with its very small spaces, might have a small low-frequency loudspeaker with limitations on the frequency range or on the power-handling capacity. Such a loudspeaker could be located in any available space without compromising the stereophonic monitoring conditions. The benefits of such an arrangement would be an improvement in the overall loudspeaker quality and some ability to adjust the low-frequency response to ameliorate the very significant problems normally encountered in such small spaces. In larger spaces, the principal benefit of such a loudspeaker system would be the ability to adjust the low-frequency response, but there would be additional small improvements in the overall monitoring quality as a result of the smaller, and less obtrusive, main loudspeakers. It is very likely that these smaller loudspeakers would also give improved stereophonic image localisation. A range of low-frequency loudspeakers could be developed to permit the selection of a suitable model for each situation. These would be relatively uncritical in design and the shape of the enclosure could be tailored to fit any reasonable space.

As part of the investigations into the use of separate low-frequency loudspeakers, a modified pair of LS5/8 loudspeakers[9] was constructed* with an internal volume of 38 litre, rather than the normal

* Loudspeaker design and development was carried out by C.D. Mathers.

110 litre. These units had all of the performance and quality attributes of the full-size models, down to a frequency of about 100 Hz. They were intended for use in subjective evaluations and additional field trials of the separate low-frequency loudspeaker arrangement. A standard LS5/8 was used for the bass unit in the tests. Although no formal results are available, early results from these tests indicate that the new loudspeakers fulfil the frequency response requirements. They also offer slightly better stereophonic imaging and are much less obtrusive.

6. DISCUSSION OF RESULTS AND CONCLUSIONS

The development of a relatively simple method for predicting low-frequency responses for near-rectangular rooms has been outlined. This method is based on the summation of modal responses and their coupling coefficients for the source and receiver positions.

Comparisons between calculated and measured responses have shown that many of the response features can be reproduced. However, some small, consistent frequency errors remained which were not evident when the modal frequencies were measured in isolation and which could not be accounted for. The method was adequate to distinguish between different room heights in a room with a ceiling structure which was difficult to interpret acoustically. It confirmed that the effects of the ceiling were frequency dependent, changing from transparent at low frequencies to reflective at higher frequencies.

A version of the prediction program was developed to produce results in one-third octave bands. This was primarily to permit comparisons with the same type of results obtained in the past from areas in which investigations of low-frequency responses had been carried out. The comparisons between these one-third octave results were, generally, less close. In many cases, only broad similarities could be discerned, with some large errors. There are two main reasons for this. Firstly, the dimensions and material data for the cases considered was incomplete — it had been recorded before the needs of the prediction method had been established. Secondly, the quantisation of the results into a small number of comparatively wide bands makes the results sensitive to the (small) errors in the frequencies of the response features. In one case, that of Dubbing Theatre W, the room construction was quite different to anything else investigated; the mode causing the greatest difficulties was far from the simply-predicted frequency, probably as a result of resonances of the steel-panelled walls.

The question of optimum room shape was also addressed. A method of illustrating the effects of room shape on the low-frequency, modal distribution was developed. This used a single criterion of room quality, the mean square mode spacing, to plot contours of 'quality' as a function of room proportions, for a number of different restrictions. From these, the regions of proportions resulting in rooms with evenly and unevenly distributed modes could be determined. Some examples have been given for rooms of fixed volume, of fixed volume and height and of fixed height only. The long-understood need to avoid rooms of square section or plan and, to a lesser extent, other simply-proportioned spaces has also been confirmed.

The frequency-response prediction and the contour plotting methods were combined to produce a method of predicting the response at any frequency for a section through a room. In the one example tested, the results were more like those measured than the output from a very expensive, Finite Element Analysis program. However, this may have been fortuitous. Also, the simple method cannot accommodate anything but the most idealistic rectangular model, whereas the FEA method can, in principle, compute the results for a model of arbitrary complexity.

The original intention of this work — the confirmation of the cause of low-frequency response irregularities — has been entirely achieved. Some progress has been made in the numerical prediction of the effects, to the extent that the results are probably limited by the uncertainties in the acoustic data on which the model is based. Because of this, there is little prospect that further progress could be made in the accuracy of the predictions for real, practical cases. It is possible that further refinement of the one-third octave prediction method would yield more accurate results than those shown here.

It is clear from this work that rooms will, inevitably, exhibit low-frequency response irregularities and that the selection of one compromise instead of another is likely to be highly subjective. The prediction method does, in principle, allow some form of preselection of room layout at the design stage. It may be possible to approach an arbitrary, but consistent, standard for control rooms of typical sizes, in order to produce at least a degree of low-frequency acoustical similarity between rooms.

7. REFERENCES

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APPENDIX

Calculations of Sound Field Pressure Levels

A.1 Reverberant sound field

The following derivations are based on Refs. 2 and 3 which represent the end-points of relatively long and complicated arguments, including many assumptions and simplifications throughout the whole book. Where earlier results are particularly relevant, the appropriate page numbers have been indicated.

From Ref. 2, the instantaneous reverberant sound pressure level, p_r, at a receiving point P (x,y,z) from a source at S (x_0,y_0,z_0) is given by:

$$p_r \approx \frac{\rho c^2 Q_0}{V} e^{-i\omega t} \sum_N \frac{\epsilon_{n_x}\epsilon_{n_y}\epsilon_{n_z} \Psi(S)\,\Psi(R)}{2\omega_N k_N/\omega + i(\omega_N^2/\omega - \omega)}$$

where

Q_0 is the volume velocity of the source,

ρ is the density of the medium,

c is the velocity of sound in the medium,

V is the volume of the room,

ω is the angular frequency,

and

ω_N is the mode natural angular frequency.

The terms ϵ_n are scaling factors depending on the order of the mode, being 1 for zero order modes and 2 for all other modes *(p. 402)*:

$$\epsilon_0 = 1,\ \epsilon_1 = \epsilon_2 = \epsilon_3 = \ldots = 2$$

The damping term, k_N, can be calculated *(p.405)* from the mode orders and the mean surface absorption coefficients. The general form of this involves a great deal of calculation relating to the mean effective pressure for different surfaces, depending on the mode order in the appropriate direction. It is simplified for rectangular rooms with 3-way uniform absorption distribution to:

$$k_N = \frac{c}{8V} \cdot \frac{(\epsilon_{n_x} a_x + \epsilon_{n_y} a_y + \epsilon_{n_z} a_z)}{2}$$

where a_x represents the total surface absorption of the room boundaries perpendicular to the x-axis, approximated by:

$$a_x = S_x \overline{\alpha}_x$$

where

S_x is the total surface area of the room boundaries perpendicular to the x-axis

and

α_x is the average absorption coefficient of the room boundaries perpendicular to the x-axis.

The functions, Ψ(x,y,z), are the three-dimensional cosine functions representing the mode spatial distributions. For the source position:

$$\Psi_N(S) = \cos\frac{n_x \pi x_0}{l_x} \cos\frac{n_y \pi y_0}{l_y} \cos\frac{n_z \pi z_0}{l_z}$$

It will be shown later that the normal type of loudspeaker produces a volume velocity inversely proportional to frequency, at least at lower frequencies. Thus, the term Q_0 in the above can be replaced by $1/\omega$

times some constant of proportionality. Assuming that this constant is unity, splitting the function into real and imaginary parts (for computational convenience) and converting to rms gives:

$$p_{r,rms} \approx \frac{\rho c^2}{\sqrt{2}\,\omega V} \sum_N \left(\frac{ab}{(b^2 + c^2)} - i \frac{ac}{(b^2 + c^2)} \right)$$

where:

$$a = \epsilon_{n_x} \epsilon_{n_y} \epsilon_{n_z} \, \Psi(S) \, \Psi(R), \quad b = 2\omega_N k_N / \omega, \quad c = \omega_N^2 / \omega - \omega$$

A.2 Direct sound field

From Ref. 3, the instantaneous direct sound pressure level, p_d, at a radial distance r from an omnidirectional source of volume velocity Q_0 is given by:

$$p_d \approx \frac{\rho}{4\pi r} Q' \left(t - \frac{r}{c} \right)$$

where the function $Q'(z)$ represents:

$$Q'(z) = \frac{d(Q(z))}{dz}$$

Substituting the usual expression for a phase-shifted sinusoidal function:

$$Q(t) = Q_0 \, e^{-i\omega\left(t - \frac{r}{c}\right)}$$

gives:

$$p_d \approx -i\omega \frac{\rho}{4\pi r} Q_0 \, e^{i\omega\left(\frac{r}{c} - t\right)}$$

Converting to rms and extracting real and imaginary terms gives:

$$p_{d,rms} \approx \frac{\omega \rho}{4\pi r \sqrt{2}} Q_0 \left(\sin \frac{\omega r}{c} - i \cos \frac{\omega r}{c} \right)$$

The magnitude of this function is proportional to frequency, which is not the usual case for normal types of loudspeakers. It is a reasonable assumption (and it can be shown) that, at low frequencies, loudspeakers produce a volume velocity inversely proportional to frequency. Replacing the term Q_0 in the above by $1/\omega$ gives:

$$p_{d,rms} \approx \frac{\rho}{4\pi r \sqrt{2}} \left(\sin \frac{\omega r}{c} - i \cos \frac{\omega r}{c} \right)$$

A.3 Total sound field

The total mean sound pressure level, p_t, is given by the sum:

$$p_t = p_r + p_d$$